Linear Programing

Define: Linear Programing is the simplest variety of programing problem in which the objective function as well as the constraint inequalities are all Known units by A.C. Chiang

Basic concept

Linear programing has two basic part–
① Objective function and
② Constraint function

① Objective function: is to maximized the profit of minimizing the cost. Thus function which is required to be maximized or minimized is called objective function.
② Structural constrained: Those quantities or expressing which state the side condition on the different activities of the problem are called structural constraint.

Feasible Region

The common region to all the constraint of Linear programing is Known as the feasible region.

Feasible Solution

The set of values of the variable which satisfies the given set of constraint and the non-negative constraint of the given LPP is the feasible solution of the given problem.

Optimum solution

The optimum solution is a feasible solution which satisfies both the structural constraint and non-negative constraint of the problem and also optimize the objective function of the problem.

Graphical Method ②

In graphical and simplex method.
It is solved geomitrically by graphing the inequalities constraints as equalities and thus determined by polygoning of feasible solution.

eg:- for example for graphical Method of LPP

Q.1. The objective function : The objective function is to maximize profit

$$\text{Max } \pi = P_1x + P_2y = 12x + 15y$$

The constraint : The three structural constraint in our problem can be written

$12x + 4y \leq 48$ ---- ①

$6x + 12y \leq 72$ ---- ②

$14x + 12y \leq 84$ ---- (3)

when x and y are two choice. Solve graphically

Note
(a) π means profit
(b) Make inequalities into equalities sign to remove smaller equal to sign.
(c) convert it into matrix form

Solution

Given $\text{Max } \pi = P_1x + P_2y = 12x + 15y$

Subjective function

$12x + 4y \leq 48$ --- (i)

$6x + 12y \leq 72$ --- (ii)

$14x + 12y \leq 84$ --- (iii)

Now make subjective function into matrix form

$$\begin{bmatrix} 12 & 4 \\ 6 & 12 \\ 14 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} 48 \\ 72 \\ 84 \end{bmatrix}$$

Non-negative $x, y \geq 0$

The graphical Method in equal sign $\leq$ and equality $=$

Now from (i) $= 12x + 4y = 48$

when $x = 0$

$12x + 4y = 48$

$12(0) + 4y = 48$

$4y = 48$

$y = \frac{48}{4} = 12$

when $y = 0$

$12x + 4y = 48$

$12x + 4(0) = 48$

$12x = 48$

$x = \frac{48}{12}$

$x = 4$

$\therefore x = 0, y = 12$
$y = 0, x = 4$

from equation (ii) $6x+12y=72$

when $x=0$

$6x+12y=72$

$6(0)+12y=72$

$12y=72$

$y=\frac{72}{12}$

$=6$

when $y=0$

$6x+12y=72$

$6x+12(0)=72$

$6x=72$

$x=\frac{72}{6}$

$=12$

$\therefore x=0,\ y=6$

$y=0\ \ x=12$

Again from equation (iii)

$14x+12y=84$

when $x=0$

$14(0)+12y=84$

$12y=84$

$y=\frac{84}{12}$

$=7$

when $y=0$

$14x+12(0)=84$

$14x=84$

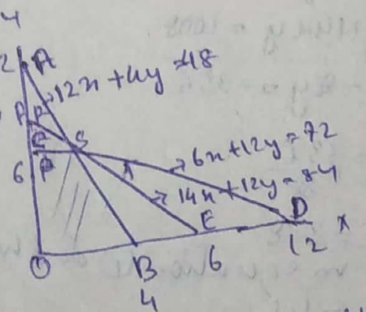
$x=\frac{84}{14}$

$=6$

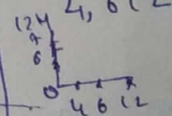
$\therefore x=0,\ y=7$

$y=0,\ x=6$

Now get feasible region using coordinates to represent a graph



Note
y coordinates are 12, 7, 6
x coordinates are 4, 6 12



Describe the graphical area of coordinates now.

feasible region shows that all the point in the shaded area bounded by the three intersecting each other would satisfying the three inequalities

At point 'S' Line EF ~~[illegible]~~ intersect line CD and at 'T' line CD intersect AB. Therefore the Shaded are is OBTSC.

Optimal solution

Out of the various point OBSTC which represent feasible region to find optimal solution

To satisfy objective function and to get π varifying, first we have to go through optimal after graphing.

continued...

(4) continue...

At point 'S' we have two equations (ii) and (iii)

$$6x + 12y = 72 \quad \ldots \text{(ii)}$$
$$\underline{-}\,14x \underline{+} 12y \underline{=} 84 \quad \ldots \text{(iii)}$$
(−)
$$-8x + 0 = -12$$
$$-8x = -12$$
$$8x = \frac{12}{8}$$
$$= 1.5 \; (3/2)$$

Now putting $x = 1.5$ in equation (ii) we get

$$6x + 12y = 72$$
$$6 \times (1.5) + 12y = 72$$
$$9 + 12y = 72$$
$$12y = 72 - 9$$
$$y = \frac{63}{12}$$
$$= 5.25$$

∴ Therefore the coordinates point $(x = 1.5,\ y = 5.25)$

Again At point 'T' we have two equations (i) and (iii)

$$12x + 4y = 48 \quad \ldots \text{(i)}$$
$$14x + 12y = 84 \quad \ldots \text{(iii)}$$

$$1 \times 14 = 168x + 56y = 672$$
$$14 \times 12 = 168x \underline{+} 144y = 1008$$

$$\text{(i)-(iii)} \quad 0 + -88y = -336$$
$$y = \frac{336}{88}$$
$$= 3.8$$

putting $y = 3.8$ in equation (i) and we have

$$12x + 4y = 48$$
$$12x + 4(3.8) = 48$$
$$12x + 15.2 = 48$$
$$x = \frac{32.8}{12}$$
$$= 2.72$$

∴ $x = 2.74,\ y = 3.8$

Now we evaluate ~~The point~~ the objective function for the coordinates point as follows

The point	Solution	Objective function $\pi = 12x + 15y$
At point C	$x = 0, y = 6$	$\pi = 12(0) + 15(6) = Rs\ 90$
At point S	$x = 1.5, y = 5.25$	$\pi = 12(1.5) + 15(5.25) = Rs\ 96.75$
At point T	$x = 2.77, y = 3.81$	$\pi = (12 \times 2.75) + 15(3.8) = Rs\ 89.91$
At point B	$x = 4, y = 0$	$\pi = 12 \times 4 + 15(0) = Rs\ 48$

∴ Rs 90, Rs 96.75, Rs 89.91 and Rs 48

H.W.

Q.2. Solve graphically

Maximize $Z = 4x_1 + 4x_2$

Subject to

$x_1 + 2x_2 \le 10$

$6x_1 + 6x_2 \le 36$

$x_1 \le 4$

where $x_1, x_2 \ge 0$.

Q.3 minimize and maximize

$Z = 5x + 10y$

Subject to

$x + 2y \le 120$

$x + y \ge 60$

$x - 2y \ge 0$

where, $x, y \ge 0$.