Mathematical Economics:

Topic

Consumer Surplus

formula

$$C\,S = \int_0^{Q} (Pd)\, dQ - P \times Q$$

Here Pd = demand function

1. eg: The consumer demand function under perfect competition is $y = 16 - x^2$ when price demand is Rs 12 and quantity $(x) = 4$

Sol. Given

$Pd = y\ 16 - x^2$

$P = 12$

$Q(x) = 4$

$\therefore$ C.S $= \int_0^{Q} (Pd)\, dQ - P \times Q$

Consumer Surplus

$$= \int_0^{4} (16 - x^2)\, dQ - 12 \times 4$$

$$= \left[\int 16 - x^2\, dx\right]_0^4 - 48$$

$$= \left[16\int - \int x^2\right]_0^4 - 48$$

$$= \left[16x - \frac{x^3}{3}\right]_0^4 - 48$$

$$= 16(4) - \frac{(4)^3}{3} - \left(16(0) - \frac{0^3}{3}\right) - 48$$

$\therefore$ By Definite integral

$$= 64 - \frac{64}{3} - 0 - 48$$

$$= \frac{192 - 64}{3} - 48$$

$$= \frac{192 - 64 - 144}{3}$$

$$= -\frac{16}{3}$$

$= -5.33$ demand.

2. Given the demand function $P = 40 - 4Q^2$, find the consumer surplus. If free goods $P = 0$.

Soln

Given $P = 40 - 4Q^2$ By integration

$P = 0$

$$C.S = \int_0^Q (P\,d) - P \times Q$$

$$P = 40 - 4Q^2$$

$$0 = 40 - 4Q^2$$

$$4Q^2 = 40$$

$$Q^2 = \frac{40}{4}$$

$$Q^2 = 10$$

$$Q = 3.2$$

$$C.S = \int_0^Q (40 - 4Q^2)\,dQ - P \times Q$$

$$= \left[40Q - 4\frac{Q^3}{3}\right]_0^{3.2} - 0 \times 3.2$$

$$= \left[40Q - \frac{4Q^3}{3}\right]_0^{3.2} - 0$$

$$= 40(3.2) - \frac{4(3.2)^3}{3} - \left(40(0) - \frac{4(0)^3}{3}\right) - 0$$

$$= 128 - \frac{4}{3} \times 32.77 - 0$$

$$= 128 - \frac{131.1}{3}$$

$$= 128 - 43.69$$

$$= 84.30$$

$\therefore$ The consumer surplus is 84.30 //

Producer Surplus

formula
$$P.S = P \times Q - \int_0^q (P_s)\, dx$$

Note:
Here P.S is based on supply side of an economy. and formula is opposite to consumer surplus. P_s = supply function

1 eg: If the market supply (function) curve is $p = 3 + x$ when 'p' and 'x' are price and quantity supply respectively. find the producer surplus).

Sol. Given $p = 5$
$P_s = 3 + x$

now
$p = 3 + x$
$5 = 3 + x$
$x = 2$

$\therefore$ Producer Surplus $(P_s) = P \times Q - \int_0^q (3+x)\,dx$

$= 5 \times 2 - \int_0^2 (3+x)\,dx$

$= 10 - \left[\int (3+x)\,dx\right]_0^2$

$= 10 - \left[3\int + \int x\right]_0^2$

$= 10 - \left[3x + \frac{x^2}{2}\right]_0^2$

$= 10 - \left\{\left(3(2) + \frac{(2)^2}{2}\right) - \left(3(0) + \frac{(0)^2}{2}\right)\right\}$

$= 10 - \left(6 + \frac{4}{2}\right) - 0$

$= 10 - 8 - 0$

$\therefore P_s = 2$ unit.

2. Given the $P = 5 + 2x$ is a market supply function, when P and x are price and supply respectively find the producer surplus when $P = 3$.

Sol. Given $p = 3$
$P = 5 + 2x$ (supply equation)

Now $P = 5 + 2x$
$3 = 5 + 2x$

$2x = -2$ $\quad$ $\therefore \frac{-2}{2} = -1$

$x = -1$

$$\therefore P.S = 3 \times (-1) - \int_0^{-1} (5 + 2x)\, dx$$

$$= -3 - \left[\int (5 + 2x)\right]_0^{-1}$$

$$= -3 - \left[5\int + 2\int x\right]_0^{-1}$$

$$= -3 - \left[5x + \frac{2x^2}{2}\right]_0^{-1}$$

$$= -3 - (5(-1) + (-1)^2) - (5(0) + 0^2)$$

$$= -3 - (-5 + 1) - 0$$

$$= -3 + 4$$

$$= 1$$

$\therefore P.S = 1.$

3. eg: Suppose the producer supply function is given by $Q = \sqrt{-5 + 3p}$ and market price is 10, find the producer surplus.

Sol: Given $P_s = Q = \sqrt{-5 + 3P}$

$P = 10.$

Now when $P = 10$ then $Q = \sqrt{-5 + 3 \times 10}$

$$= \sqrt{-5 + 30}$$

$$= \sqrt{25}$$

$$Q = 5$$

Again to obtain producer equation by squaring both side.

$$Q = \sqrt{-5 + 3P}$$

$$Q^2 = -5 + 3P$$

$$\therefore P = \frac{5}{3} + 3Q^2$$

$$\therefore P_s = P \times Q - \int_0^5 (P_s)\, dx$$

$$= 10 \times 5 - \int_0^5 \left(\frac{5}{3} + 3Q^2\right) dx$$

$$= 50 - \left[\int \frac{5}{3} + 3Q^2\right]_0^5$$

$$= 50 - \left[\frac{5}{3}\int + 3\int Q^2\right]_0^5$$

$$= 50 - \left[\frac{5}{3}Q + Q^3\right]_0^5$$

$$= 50 - \left(\frac{5}{3}(5) + (5)^3\right) - \left(\frac{5}{3}(0) + (0)^3\right)$$

$$= 50 - \left(\frac{25}{3} + 125\right) - 0$$

$$= 50 - \frac{25}{3} - 125$$

$$= \frac{150 - 25 - 375}{3}$$

$$= -\frac{250}{3}$$

$$= -83$$

$\therefore$ Producer Surplus is -83 //

Capital formation

formula

$$\frac{dk}{dt} = I(t)$$

$$\therefore K(t) = \int_0^4 I(t)\,dt$$

Note:
$I(t)$ investment in period of time,
K = constant
By integration

eg: If $I(t) = 2t$ crores of rupees per year. What will be the capital formation in the time period of 4 years and also in 4th year.

Notes
Here we have to calculate for 2 years. i.e.
(i) upto 4 years
(ii) last 4th year

Sol. given

$$It = 2t \text{ crore},$$

$$CF\ I(t) = \int_0^4 2(t)\,dt$$

$$= \left[2\int t\,dt\right]_0^4$$

$$= \left[\frac{2t^2}{2}\right]_0^4$$

$$= \left[t^2\right]_0^4$$

$$= 4^2 - 0^2$$

$$= 16$$

$\therefore$ For the 4 years of capital formation is 16 crore.

Again for the

Pg. 4.

Again for the 4th year.

Here lower limit is 3 from last year

By definite integral

$$I(t) = \int_3^4 2t\, dt$$

$$= \left[\int 2t\, dt\right]_3^4$$

$$= \left[2 \cdot \frac{t^2}{2}\right]_3^4$$

$$= \left[t^2\right]_3^4$$

$$= 4^2 - 3^2$$

$$= 16 - 9$$

$$= 7 \text{ crore}$$

$\therefore$ The capital formation for the 4th year is 7 crore.