Matrix

Cramer Rule

① Solve the equation using Cramer Rule

$$3x + y - z = 5$$
$$-2x + 3y - 2z = 2$$
$$4x - 2y + 3z = 4$$

Sol.

Step 1. convert into Square matrix, eg. A=11, X=11

$$\begin{bmatrix} 3 & 1 & -1 \\ -2 & 3 & -2 \\ 4 & -2 & 3 \end{bmatrix}_{3\times 3} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 4 \end{bmatrix}$$

A X C

Step 2. Find |A| taking first Row elements as, |A| = + − +

$$|A| = 3\begin{vmatrix} 3 & -2 \\ -2 & 3 \end{vmatrix} - 1\begin{vmatrix} -2 & -2 \\ 4 & 3 \end{vmatrix} + (-1)\begin{vmatrix} -2 & 3 \\ 4 & -2 \end{vmatrix}$$

$$= 3(3\times 3 - (2\times -2)) - 1(-2\times 3 - (2\times 4)) - 1(-2\times 2 - 4\times 3)$$
$$= 3(9-4) - (-6+8) - 1(-4-12)$$
$$= 3(5) - (2) - 1(-16)$$
$$= 15 - 2 + 16$$
$$|A| = 29$$

Step 3

Now exchange the C square elements value with first column of A matrix to get $|X|$ =

$$|X| = \begin{bmatrix} 5 & 1 & -1 \\ 2 & 3 & -2 \\ 4 & -2 & 3 \end{bmatrix}$$

$$|X| = 5\begin{vmatrix} 3 & -2 \\ -2 & 3 \end{vmatrix} - 1\begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix}$$
$$= 5(9-(4) - 1(6-8) - 1(-4-12)$$
$$= 5(5) - 1(-2) - 1(-16)$$
$$= 25 + 2 + 16$$
$$|X| = 43$$

step 4 & 5 are same method as with step 3.

$$|Y| = \begin{bmatrix} 3 & 5 & -1 \\ -2 & 2 & -2 \\ 4 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -2 & 2 \\ 4 & 4 \end{bmatrix}$$

$$|Y| = 3\begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} - 5\begin{vmatrix} -2 & -2 \\ 4 & 3 \end{vmatrix} + (-1)\begin{vmatrix} -2 & 2 \\ 4 & 4 \end{vmatrix}$$
$$= 3(6+8) - 5(-6+8) - 1(-8-8)$$
$$= 3(14) - 5(2) - 1(16)$$
$$= 42 - 10 - 16$$
$$= 42 - 26$$
$$|Y| = 16$$

cont. 1.

$$Z = \begin{bmatrix} 3 & 1 & 5 \\ -2 & 3 & 2 \\ 4 & 2 & 4 \end{bmatrix}$$

$$|Z| = 3\begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} - 1\begin{vmatrix} -2 & 2 \\ 4 & 4 \end{vmatrix} + 5\begin{vmatrix} -2 & 3 \\ 4 & 2 \end{vmatrix}$$

$$= 3(12-4) - 1(-8-8) + 5(-4-12)$$

$$= 3(8) - 1(16) + 5(-16)$$

$$= 24 - 16 - 80$$

$$= 24 - 96$$

$$= -72$$

Now in last step 6 divide all x, y, z determinant by $|A|$. $\frac{|X|}{|A|}$

$$x = \frac{|X|}{|A|} = \frac{43}{29} = 1.48$$

$$y = \frac{|Y|}{|A|} = \frac{16}{29} = 0.55$$

$$z = \frac{|Z|}{|A|} = \frac{-72}{29} = -2.48$$

i.e. $x = 1.48$, $y = 0.55$, $z = -2.48$

$\therefore |X| = 1.48, |Y| = 0.55, |Z| = -2.48, |A| = 29$

H.W

eg. 2. Try this

Solve the following equation using Cramer Rule.

$$3x + y = 1$$
$$-2x + 3y + 4z = 2$$
$$+4y - 3z = 3$$

Sol:

$$\begin{bmatrix} 3 & 1 & 0 \\ -2 & 3 & 4 \\ 0 & 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

A X C

eg. 3 Solve by Cramer Rule

$$2x + 7y - 5 = 0$$
$$-3x + 8y = -11$$

Sol. Now

$$2x + 7y = 5$$
$$-3x + 8y = -11$$

$$\begin{bmatrix} 2 & 7 \\ -3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -11 \end{bmatrix}$$

A (2×2) X C

$$|A| = \begin{bmatrix} 2 & 7 \\ -3 & 8 \end{bmatrix} = (2 \times 8 - 3 \times 7) = 16 - (-21) = 16 + 21 = 37$$

contd. 2

$$|X| = \begin{bmatrix} 5 & 7 \\ -11 & 8 \end{bmatrix} = (5\times 8 - 7\times(-11)) = (40+77) = 117$$

$$|Y| = \begin{bmatrix} 2 & 5 \\ -3 & -11 \end{bmatrix} = (2\times(-11) - 5\times(-3)) = (-22+15) = 7$$

$$x = \frac{|X|}{|A|} = \frac{117}{37} = 3.14$$

$$y = \frac{|Y|}{|A|} = \frac{7}{37} = 0.19$$

Eg. 4

Solve by cramer Rule

$$0.x + 2z = 3$$

$$+0.2y + z = 3$$

$$0.3x + 0.4y = 5$$